Middle School Mathematics and the Common Core Institute Summer 2012

Faculty Name: Debbie Morris
School: Baxterville School
Grade Level: 6th Grade

1. Teaching Objective

- **Common Core State Standards of Mathematics**
  6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- **2007 MS Mathematics Framework Revised**
  2. Use algebraic functions, patterns, and language across a variety of contexts.
    c. Formulate algebraic expressions, equations, and inequalities to reflect a given situation. (DOK 2)

** Given real-world problems or situations, TSW use variables to formulate equations through the use of patterns with 80% accuracy.

2. Instructional Activities

☀ Anticipatory Set: TTW demonstrate the following problem: Suppose I tear a piece of paper in half and give half to someone else. (TTW actually do this.) Then, each of us tears our piece in half and passes half on to someone else. How many pieces of paper are there distributed now? After two rounds we have distributed 4 pieces of paper. The teacher and students with paper will again tear their pieces of paper in half and give half to someone else.

TTW ask the following questions:

- Our paper pieces are already getting small, but without tearing them any further, how many rounds of tears do you think it would take for everyone in this room to have a piece of paper?
- If there are 36 students in sixth grade at our school, how many rounds of tears do you think it would take for every student in the sixth grade to have a piece of paper?
- If there are 326 students in our whole school, how many tears would it take for every student in the school to have a piece of paper?
- Can you think of a way to solve this problem without having to tear these pieces of paper into microscopic pieces?
Modeling/Guided Practice: TTW listen to the students’ ideas as to how to solve the problem without physically tearing the paper. If no student comes up with the idea of finding a pattern, TTW suggest that a student volunteer create a table on the promethean board as follows:

<table>
<thead>
<tr>
<th>Rounds of Tears (x)</th>
<th>Pieces of Paper (y)</th>
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</thead>
<tbody>
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TTW will distribute the attached handout entitled, “Share and Share Alike.” TSW work in groups to complete the handout. TTW assist students as needed by giving clues to answering questions. TTW randomly select a student to answer each question from the handout and record the information on the board.

TTW have the students give a thumbs up if they thought they understood the problem or a thumbs down if they were struggling. During independent work, TTW work individually or in small groups with students struggling to understand the concept.

Independent Practice: Congratulations!! You did such a great job on using a pattern to solve our torn paper dilemma that now Coach has put you in charge of setting up the tables for the athletic banquet. Please use the colored tiles at your table to assist you in completing the “I Need My Space” handout.

3. Materials and Resources

Materials
- Sheet of typing paper (to tear into pieces)
- Colored Tiles
- Promethean Board
- “Share and Share Alike” (Teacher Created Handout)
- “I Need My Space” (Teacher Created Handout)

Resources
4. **Assessment**

“Share and Share Alike” and “I Need My Space” handouts will be graded for formal assessments.
Students will be informally assessed during task performance in creating colored tile banquet table models and in group participation.
### Share and Share Alike

<table>
<thead>
<tr>
<th>Rounds of Tears (x)</th>
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</tbody>
</table>

**Answer these questions to help you fill in the chart.**

1. After the first tear, how many pieces of paper did we have? ______________

2. After the second round of tears, how many pieces of paper did we have? ________
   - The third round of tears? ________  
   - The fourth round of tears? ________  
   - The fifth round of tears? ________

3. At least how many rounds of tears would be needed in order for each of the 18 students in this class to receive a piece of paper? ________________________________

   How many pieces would be left over? (Show your work.)
   __________________________________________________________________________

4. Every time the round of tears increases by 1, what happens to the number of pieces?
   __________________________________________________________________________
   __________________________________________________________________________

5. What pattern can you formulate using the information given in the table? (Hint: Think about exponents.)
   __________________________________________________________________________
   __________________________________________________________________________
   __________________________________________________________________________
6. Write an equation containing a variable to explain the pattern for the table. (Use $x$ to represent the rounds of tears and $y$ to represent the pieces of paper.) Justify that your equation works.

_____________________________________________________________________

7. Using your equation, at least how many rounds of tears would it take for each of the 36 students in sixth grade to receive a slip of paper? (Show your answer and justify it by using your equation.) ________________

How many pieces of paper would be left over? (Show your work) ________________

8. Using your equation, at least how many rounds of tears would it take for each of the 326 student in the school to receive a slip of paper? (Show your answer and justify it by using your equation.) ________________

How many pieces of paper would be left over? (Show your work) ________________
Share and Share Alike (Answer Key)

<table>
<thead>
<tr>
<th>Rounds of Tears (x)</th>
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<tbody>
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<td>2</td>
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<tr>
<td>3</td>
<td>8</td>
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<tr>
<td>4</td>
<td>16</td>
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<tr>
<td>5</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>128</td>
</tr>
</tbody>
</table>

Answer these questions to help you fill in the chart.

1. After the first tear, how many pieces of paper did we have? _____ 2 _____

2. After the second round of tears, how many pieces of paper did we have? _____ 4 _____
   The third round of tears? _____ 8 _____ The fourth round of tears? _____ 16 _____ The fifth round of tears? _____ 32 _____

3. At least how many rounds of tears would be needed in order for each of the 18 students in this class to receive a piece of paper? 5 rounds of tears which would result in 32 pieces of paper

   How many pieces would be left over? (Show your work)
   \[
   32 - 18 = 14, \text{ so 14 pieces of paper would be left over}
   \]

4. Every time the round of tears increases by 1, what happens to the number of pieces?
   Every time the round of tears increases by one, the number of pieces doubles from the previous number of pieces.

5. What pattern can you formulate using the information given in the table?
   (Hint: Think about exponents.) Students should notice that the pattern for the number of pieces of paper doubles each time. If necessary, lead students in concluding that since for each round that the paper is torn, the number of pieces of paper doubles, then if you say two to the power of the round, you will get the number of pieces of paper that you have.
6. Write an equation containing a variable to explain the pattern for the table. (Use $x$ to represent rounds of tears and $y$ to represent pieces of paper.) Justify your equation.

$y = 2^x$ To justify that:  
Round 1  $2 = 2^1$  
Round 2  $4 = 2^2$  
Round 3  $8 = 2^3$  
Round 4  $16 = 2^4$  
Round 5  $32 = 2^5$  

7. Using your equation, at least how many rounds of tears would it take for each of the 36 students in sixth grade to receive a slip of paper? (Show your answer and justify it by using your equation.) Any reasonable justification will be accepted. Students should be able to formulate something like this: $2^5 \geq 36$  
$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$ (not quite enough)  
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8 = 64$ (more than enough)  
Therefore, it would take 6 rounds of tears for every student in sixth grade to receive a slip of paper.

How many pieces of paper would be left over? (Show your work)  
$64 - 36 = 28$, so 28 pieces of paper would be left over.

8. Using your equation, at least how many rounds of tears would it take for each of the 326 students in the school to receive a slip of paper? (Show your answer and justify it by using your equation.) Any reasonable justification will be accepted. Students should be able to formulate something like this: $2^9 \geq 326$  
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9 = 512$  
It would take at least 9 rounds of tears in order for each of the 326 students to receive a piece of paper.

How many pieces of paper would be left over? (Show your work)  
$512 - 326 = 186$  
There would be 186 pieces of paper left over.

Grading Rubric for Share and Share Alike  
(Total Points -- 50)

Answers to Table -- 14 answers at ½ point each  
1 point
Question 1 -- 1 fill in the blank  
2 points
Question 2 -- 4 fill in the blanks at ½ point each  
3 points for justification/explanation  
5 points
Question 3 -- 2 points at 1 point for each question  
5 points
Question 4 -- explanation  
5 points
Question 5 -- explanation  
5 points
Question 6 -- 2 points for equation  
3 points for reasoning/justification  
5 points
Question 7 -- 2 answers at 2 points each  
2 explanations/justifications at 3 points each  
10 points
Question 8 -- 2 answers at 2 points each  
2 explanation/justifications at 3 points each  
10 points

Total  
50 points
I Need My Space

Congratulations!! You have just been selected to serve on the committee to set up the tables for the athletic banquet. One square table seats four people. However, in order for everyone to be able to see the end-of-the-year slide show, Coach has asked that your committee set up the tables in one long row. Use your colored tiles to represent the tables. Create a chart similar to the one you used in “Share and Share Alike.” Then, work as a group to use your problem solving skills to answer the questions below.

<table>
<thead>
<tr>
<th>Tables (x)</th>
<th>Number of People (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

1. Complete the chart. Use the pattern from the colored tiles to determine the number of people that will be able to sit at 5 tables. Justify or explain how you found your answer. __________________________________________
   ______________________________________________________________
   ______________________________________________________________
   ______________________________________________________________

2. If you set up 23 tables, will there be enough seating for at least 50 people? Justify your answer. ___________________________________________
   ______________________________________________________________

3. On a clean sheet of paper, create a question of your own about the table arrangement. Determine the answer but don’t write it on the paper.

4. Get out another clean sheet of paper. Exchange your question with the members of your group. Write down and solve each group members’ problems. The author of the problem will act as the teacher to check to see if you correctly solved his/her problem.
I Need My Space  (Answer Key)

Congratulations!! You have just been selected to serve on the committee to set up the tables for the athletic banquet. One square table seats four people. However, in order for everyone to be able to see the end-of-the-year slide show, Coach has asked that your committee set up the tables in one long row. Use your colored tiles to represent the tables. Create a chart similar to the one you used in “Share and Share Alike.” Then, work as a group to use your problem solving skills to answer the questions below.

1. Complete the chart. Use the pattern from the colored tiles to determine the number of people that will be able to sit at 5 tables. Justify or explain how you found your answer. Twelve people would be able to sit at 5 tables. Students can explain by illustrating with their colored tiles or by showing that after the first table, the number of people increases by 2 for each table added.

<table>
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<td>8</td>
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<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

2. If you set up 23 tables, will there be enough seating for at least 50 people? Justify your answer. No. Twenty-three tables would only seat 48 people. Students may justify this by a variety of methods. They may model 23 tables using their colored tiles. They may use their chart to illustrate the information, or they may come up with a method of their own. Once students explain their answers, challenge them to formulate an equation to justify their information. The equation for this pattern is \( y = x + (x + 2) \) or \( y = 2x + 2 \).
Grading Rubric for I Need My Space  
(Total Points -- 20)

Answers to Table -- 10 answers at $\frac{1}{2}$ point each  
Question 1 – 2 points answer  
  3 points for justification/explanation  
Question 2 – 2 points answer  
  3 points for justification/explanation  
Question 3 – formulated problem and solution  

Total  

5 points  
5 points  
5 points  
5 points  
20 points
Checklist for “I Need My Space” Activity

<table>
<thead>
<tr>
<th>Name</th>
<th>Active Participation</th>
<th>Appropriate Use of Manipulatives in Solving Problem</th>
<th>Provided Oral Explanation and Justification of the Group Strategy for Solving the Problem</th>
<th>Comments</th>
<th>Date ________________</th>
</tr>
</thead>
</table>

***Note √ indicates student completed objective; X indicates students did not complete objective.